Use the following values for constants.

Acceleration due to gravity on Earth	g	10.00 m·s^{-2}
Boltzmann constant	k_B	1.38×10^{-23} J·K^{-1}
Current Mass of the Sun	M_s	2.00×10^{30} kg
Current Radius of the Sun	R_s	7.00×10^{8} m
Magnitude of the electron charge	e	1.60×10^{-19} C
Mass of the electron	m_e	9.11×10^{-31} kg
Mass of the proton	m_p	1.67×10^{-27} kg
Atomic Mass Unit	u	931.50 MeV/c^2
Permeability of free space	μ_0	1.26×10^{-6} H·m^{-1}
Permittivity of free space	ϵ_0	8.85×10^{-12} F·m^{-1}
Planck's constant	h	6.63×10^{-34} J·s
Avogadro Constant	N_A	6.02×10^{23} mol^{-1}
Speed of light in vacuum	c	3.00×10^{8} m·s^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W·m$^{-2} \cdot$ K^{-4}
Universal Gas constant	R	8.31 J·K$^{-1} \cdot$ mol^{-1}
Universal Gravitational constant	G	6.67×10^{-11} kg^{-1}·m$^{3} \cdot$ s^{-2}
Wien's constant	b	2.90×10^{-3} m·K
$\ln 2$	$\approx$	0.69
$\ln 3$	$\approx$	1.10
$\ln 10$	$\approx$	2.30
Base of the Napierian logarithm e	$\approx$	2.72

SET A

- The first 12 questions are multiple choice questions with only one answer correct. The candidate gets 2.5 marks for a correct answer and -1 for an incorrect answer.
- Questions $13-17$ are multiple choice questions and more than one answer might be correct. The candidate gets 4 marks for selecting **all** the correct answers. There is no negative marking.

1. A thin spherical copper shell of radius R, completely filled with a viscous fluid, is rotating about the vertical axis with a constant angular speed ω_0. Due to a leak at the bottom of the shell, the fluid starts dripping steadily and vertically from the shell. The net change in angular speed ($\delta\omega$) when the shell gets empty is

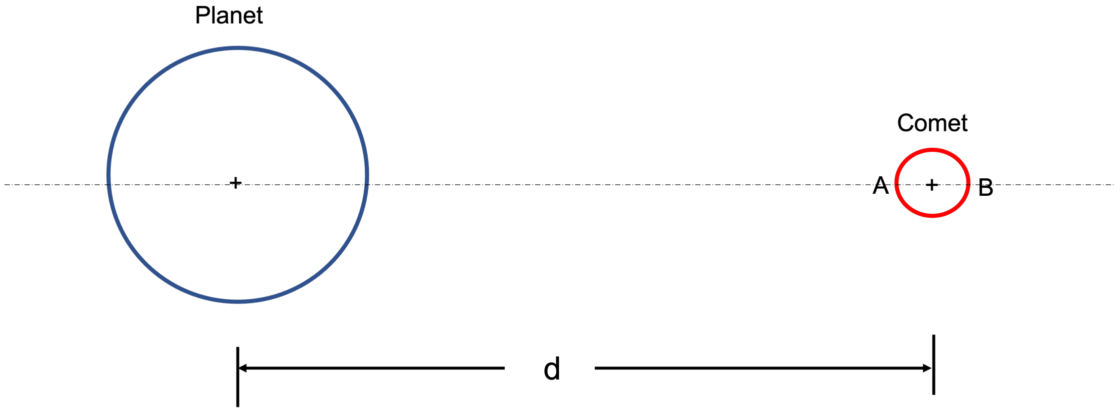
   A. proportional to R^2.

   B. proportional to R.

   C. proportional to R^3.

   D. independent of R.

2. A spherical comet having mass M_s and radius r is moving towards a planet of mass M_p as shown in the figure. At a separation distance d, equal gravitational force is experienced by the two identical test masses (m) which are placed at diametrically opposite ends (A and B) of the comet. Assuming $d \gg r$, the correct choice about the separation distance d is,



A. d is proportional to $M_p^{1/3}$.

B. d is proportional to $M_s^{2/3}$.

C. d is independent of r.

D. d is inversely proportional to $M_s^{2/3}$.

3. A metal rod, connected between two high voltage electrodes, attains steady-state temperature through a balance between radiated power loss and Joule heating. The temperature of the surrounding is neglegible compared to that of the rod and the resistance of the rod is independent of its temperature. Assuming the current through the rod to be I, the dominant wavelength of radiation(λ) is given by $\lambda \propto I^{\alpha}$. Then, the value of α is

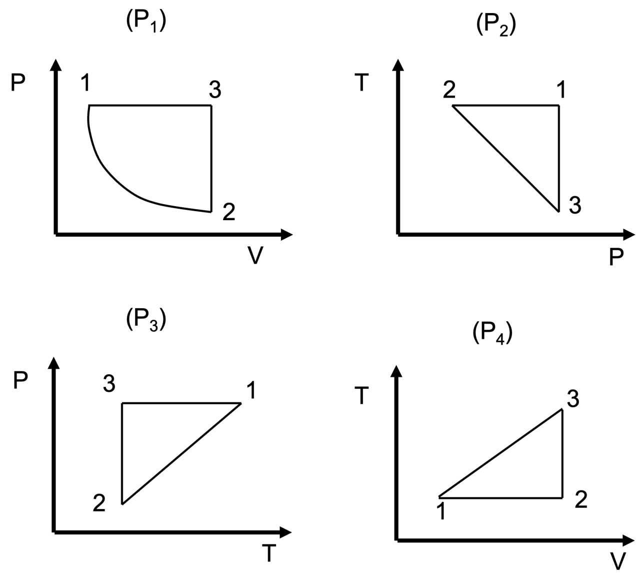
   A. 1.0

   B. -0.5

   C. 2.0

   D. -1.0

4. Thermodynamic processes (P_1,P_2,P_3,P_4) in which an ideal gas passes through states 1, 2 and 3 are shown in the figure where P, V, T are pressure, volume and temperature, respectively . The process(processes) that could be identical to the process P_1 is (are)



A. P_2 only.

B. P_2 and P_3 only.

C. P_3 and P_4 only.

D. P_2 and P_4 only.

5. A proton accelerated from rest by a potential difference of V volts has a de Broglie wavelength of 0.20Å (1.0 Å $=10^{-10}$ m). A fully ionized Helium atom is similarly accelerated by a potential difference of 2 V volts. Its de Broglie wavelength (Å) is closest to

   A. 0.05

   B. 0.07

   C. 0.10

   D. 0.20

6. Consider the Bohr model of the hydrogen atom with Bohr radius a_B. If the mass of the electron and that of the proton become twice the present values, then the new Bohr radius will

   A. remain unchanged.

   B. change to $2a_B$.

   C. change to $a_B/2$.

   D. change to $4a_B$.

7. Two radioactive samples X and Y have the same number of atoms initially $[N_X(t = 0) = N_Y(t = 0)]$. The half life $\tau^x_{1/2}$ of X is half the mean life of Y. Then $N_Y(t)$ is seven times $N_X(t)$ when $t/\tau^x_{1/2}$ is closest to

   A. 1

   B. 2

   C. 5

   D. 10

8. An elastic conducting ring of mass m is extended radially with constant speed v in an uniform magnetic field of strength B which is perpendicular to the plane of the ring. Take the resistance R of the ring to be a constant. The magnetic moment (μ) of the ring in terms of the instantaneous radius of the ring (r) is given by $\mu = Kr^{\alpha}$ where K and α are constants. Then, the value of α is

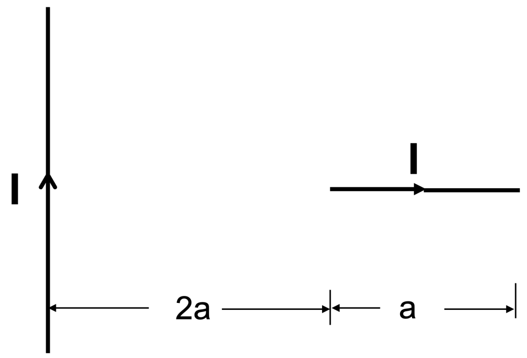
   A. 0

   B. 1

   C. 4

   D. 3

9. A horizontal straight wire of length a is placed perpendicular to a long current carrying straight vertical wire at a distance of $2a$ and lies in the same plane as shown in the figure. Both wires carry steady current I. The magnitude of the force on the horizontal wire due to the vertical wire is



A. $\dfrac{\mu_0 I^2 \ln(3/2)}{2\pi}$

B. 0

C. $\dfrac{\mu_0 I^2 \ln 3}{\pi}$

D. $\dfrac{3\mu_0 I^2 \ln 2}{2\pi}$

10. A double convex lens of the objective is changed to plano-convex. The objective is made of a plastic material with refractive index 1.3. Then,

    A. the numerical aperture of the double convex lens is 1.3.

    B. the numerical aperture of the plano-convex lens is 0.39.

    C. the diameter to focal length ratio of the plano-convex lens is 1.2.

    D. the critical angle of the double convex lens is 60°.

11. An elastic wave generates a stress of magnitude N while propagating in a wire. The relation between its frequency ω and the wavevector k is given by $\omega = \omega_0\sqrt{1-\cos ka}$, where ω_0 iand a are constants. In the long wavelength approximation $(\lambda \gg a)$, the linear density of the wire is

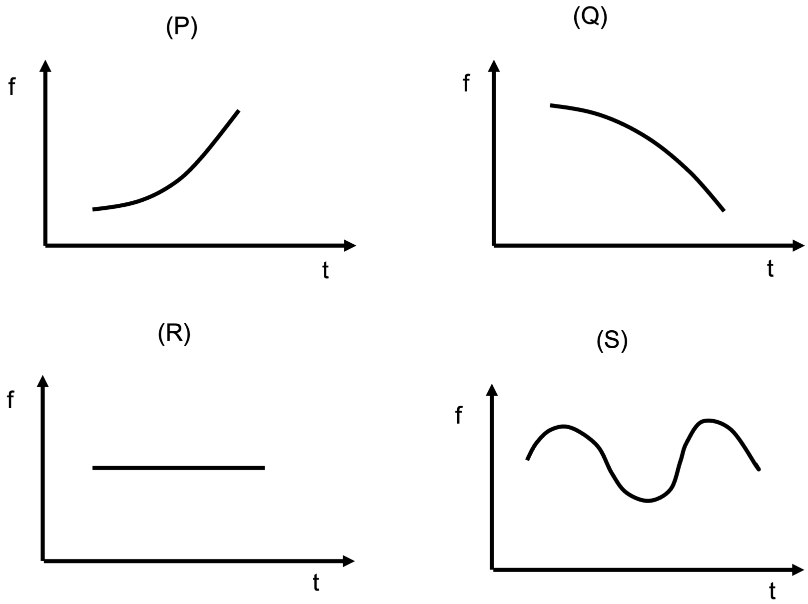
A. $2N/\left(\omega_0 a\right)$

B. $\omega_0 a/N$

C. $2N/\left(\omega_0 a\right)^2$

D. $N/\left(\omega_0^2 a\right)$

12. A slow steady stream of water is falling onto a tall cylindrical bucket. Let $f(t)$ denote the dominant frequency of the sound of the fall onto the horizontal water surface in the bucket. The possible time dependences of this frequency are shown in the figures. The graph which best describes the variation of f with t qualitatively is



A. P

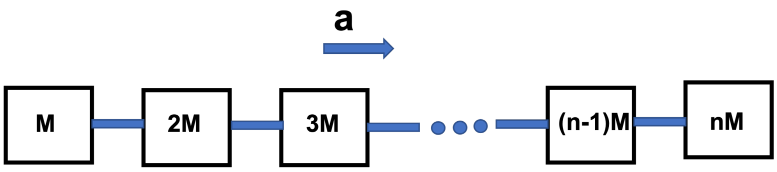
B. Q

C. R

D. S

The following questions may have more than one correct answer. Please select all the correct answers.

13. An accelerating train (acceleration $a = 1$ m$\cdot$s^{-2}) of n blocks have a mass distribution as shown in the figure. The rightmost block of mass nM is the engine. The blocks are connected through a Aluminum cable of cross-section 10 cm^2. The maximum allowed strain in the connecting cables is 0.001. Taking $M = 1000$ kg and Young's modulus of Aluminum to be 7×10^{10} Pa, the correct choice(s) is(are).



A. all connecting cables will have equal strain.

B. the difference of stress in any two consecutive cables is in arithmetic progression.

C. the train can have at most 12 blocks.

D. the maximum stress is between blocks having masses M and $2M$.

14. A point object P of mass m and charge q is placed at a distance r from a stationary object Q of mass M and charge $-q$. Let T_g be the time for P to reach Q if it is moving only under gravitational attraction. Similarly, let T_e be the time for P to reach Q if it is moving only under electrostatic attraction. Then,

    A. T_g depends on M but not on m.

    B. T_e depends on m but not on M.

    C. T_g^2 is directly proportional to G.

    D. T_e^2 is directly proportional to ϵ_0.

15. The correct statement(s) about an ideal gas is(are)

    A. for an adiabatic process, the work-done does not depend upon the path.

    B. for an isobaric process, the change in enthalpy is equal to the net heat input to the system.

    C. the total potential energy of an ideal gas can be a function of time.

    D. for an isothermal process, the entropy of an ideal gas remains constant.

16. The electrostatic potential in the region between two long coaxial cylinders of radii a and b is given by $\phi = \alpha \ln(r/a) + \beta$, where α and β are constants. Here, $\alpha > 0$ and r denotes radial distance from the axis such that $a < r < b$. Then, the correct option(s) is (are)

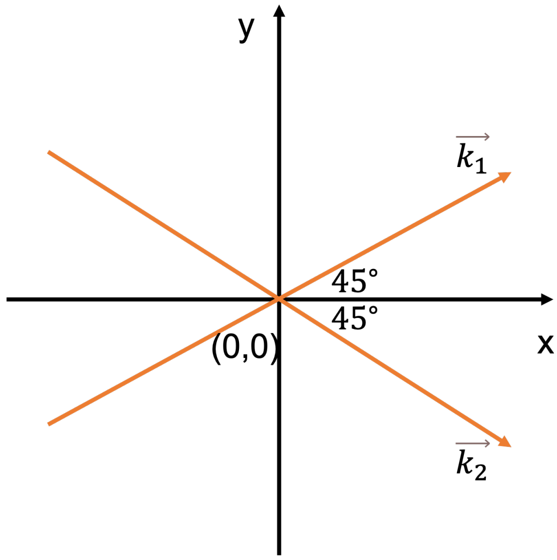
    A. The charge per unit length on the inner cylinder is $-(2\pi\epsilon_0\alpha)$.

    B. Capacitance per unit length is $\ln(b/a)/(2\pi\epsilon_0)$.

    C. The charge density on the outer cylinder is $-(\epsilon_0\alpha)/b^2$.

    D. The electric field in the region between two cylinders is $-(\alpha/r)\hat{r}$.

17. Two plane waves having amplitude E_0 are described by $\vec{E_1} = E_0 \cos\left(\vec{k_1} \cdot \vec{r} - \omega t\right) \hat{z}$ and $\vec{E_2} = E_0 \cos\left(\vec{k_2} \cdot \vec{r} - \omega t\right) \hat{z}$. The wavevectors $\vec{k_1}$ and $\vec{k_2}$ pass through the origin making an angle of $45°$ with x-axis, as shown in the figure. At $t = 0$ and $x = 0$ plane, the correct option(s) is(are)



A. the periodicity of interfence pattern in the yz plane is $\sqrt{2}\lambda$.

B. the amplitude of resultant wave is $2E_0$.

C. the direction of polarization of resultant wave will change.

D. the maximum intensity is E_0^2.