1. Let $g : \mathbb{R} \mapsto \mathbb{R}$ be a differentiable function such that $g(x)g'(x) > 0$ for all $x \in \mathbb{R}$. Then

   (a) g is increasing.

   (b) g is decreasing.

   (c) $|g|$ is increasing.

   (d) $|g|$ is decreasing.

2. The number of real roots of $f(x) = x^6 + x^3 - 1$ is

   (a) 0

   (b) 2

   (c) 4

   (d) 6

3. In a throw of a (biased single) dice, the probability of the outcome being a number n is $\frac{1}{4}$ if n is even, and $\frac{1}{12}$ if n is odd. If the dice is thrown twice, then the probability that the sum of the two outcomes is an even number is

   (a) $\dfrac{3}{8}$

   (b) $\dfrac{1}{2}$

   (c) $\dfrac{5}{8}$

   (d) $\dfrac{3}{4}$

4. Define $\operatorname{sgn}(x) = \begin{cases} 1, & \text{if } x > 0, \\ -1, & \text{if } x < 0, \\ 0, & \text{if } x = 0. \end{cases}$

   Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = (x - \sqrt{5}) \operatorname{sgn}(x^2 - 5)$. Then the number of discontinuities of f is

   (a) 0

   (b) 1

   (c) 2

   (d) 3

5. Let S be the set of all natural numbers x such that

(i) $100 \leq x \leq 999$,

(ii) 0 appears at least once as a digit in the decimal expansion of x, and

(iii) the sum of the digits of x is 10.

Then the number of elements in S is

(a) 18
(b) 20
(c) 27
(d) 30

6. The horizontal line $y = k$ intersects the parabola $y = 2(x-4)(x-6)$ at points A and B. If the length of AB is 8, then the value of k is

(a) 30
(b) 10
(c) 20
(d) 8

7. Let $S(n) = \frac{1}{n^4}\sum_{l=1}^{n}(l+2)(l+4)(l+6)$. The value of $\lim_{n\to\infty} S(n)$ is

(a) $\frac{1}{6}$
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) 1

8. Let α be a complex number such that $\alpha \neq 1$ and $\alpha^5 = 1$. Let $A = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \alpha & 0 \\ \alpha & 0 & 0 \end{pmatrix}$ and I denote the identity matrix. Then the value of $I + A + A^2 + A^3 + A^4$ is

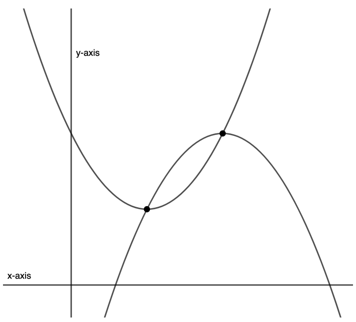
(a) $(1+\alpha^2+\alpha^4)\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

(b) $\alpha(1+\alpha^2)\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

(c) $(1+\alpha^2+\alpha^4)\begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

(d) $(1+\alpha^2+\alpha^4)\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$

9. Let P and Q be the vertices of the parabolae $y = x^2+bx+c$ and $y = -x^2+dx+e$, respectively.



If P and Q are the points of intersection of the parabolae then the slope of the line through P and Q is

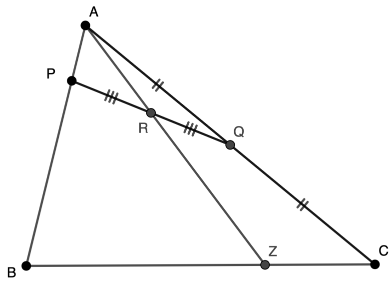
(a) $\dfrac{c+e}{2}$

(b) $\dfrac{c+d}{2}$

(c) $\dfrac{b+d}{2}$

(d) $\dfrac{b+e}{2}$

10. Let ABC be a triangle with $AC = 2048$, $AB = 512$ and $BC = 2000$. Let P be a point on the segment AB such that $AP = 1$ and Q be a point on the segment AC such that $AQ = 1024$. Let R be the midpoint of PQ. Let Z be the point of intersection of AR and BC.



Then the length of ZC is

(a) $\dfrac{2000}{256}$

(b) $\dfrac{2000}{257}$

(c) $\dfrac{1000}{256}$

(d) $\dfrac{1000}{257}$

11. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(0) = 1$ and

$$|f(x) - f(y)| \leq |\sin\{(x-y)^2\}| \text{ for all } x, y \in \mathbb{R},$$

and let g be the function defined by $g(x) = x^2 f(x^2)$ for all $x \in \mathbb{R}$. Then the value of $g'(2)$ is

(a) 2

(b) 4

(c) 6

(d) 0

12. Let $n \geq 3$ be an integer. Let $P_1, P_2, \ldots, P_{2n}$ be points in the plane, which are the vertices of a regular $2n$-gon. The number of obtuse-angled triangles with vertices contained in the set $\{P_1, P_2, \ldots, P_{2n}\}$ is

   (a) $n(n-1)(n-2)$

   (b) $\dfrac{n^2(n-1)(n-2)}{3}$

   (c) $\dfrac{n(n-1)^2}{2}$

   (d) $2n(2n-1)(2n-2)$

13. If A, B, C are 3×3 matrices with entries in $\mathbb{R}$, satisfying the condition $AB = AC$, then

   (a) the determinant of AB is 0.

   (b) either A is the zero matrix or $B = C$.

   (c) either $B = C$ or A is not an invertible matrix.

   (d) either A is the zero matrix or the determinant of $B - C$ is zero.

14. Let X, Y, Z be sets and $f\colon X \to Y$ and $g\colon Y \to Z$ be functions. Then

   (a) $g \circ f$ being injective implies f injective.

   (b) $g \circ f$ being surjective implies g surjective.

   (c) $g \circ f$ being injective implies g injective.

   (d) g being surjective implies $g \circ f$ surjective.

15. Let $f : (0,3) \cup (6,9) \to \mathbb{R}$ be a differentiable function such that $f'(x) = \frac{1}{2}$ for all $x \in (0,3) \cup (6,9)$. Then

   (a) f is an increasing function.

   (b) f is a one to one function.

   (c) $f(8) - f(7) = f(2) - f(1)$.

   (d) there exists a number c in $\mathbb{R}$ such that $f(x+6) = f(x)+c$ for all $x \in (0,3)$.

16. Let A and B be two points on the parabola $y - 2x^2 = 0$ and O be the origin $(0, 0)$. If

   (a) OAB is an isosceles triangle then the y coordinates of A and B are equal.
   (b) OAB is an equilateral triangle then the length of each side is $\sqrt{3}$.
   (c) OAB is an isosceles triangle and the two equal sides are of length $\sqrt{3}$ then OAB is an equilateral triangle.
   (d) OAB is an equilateral triangle then its altitude is $\sqrt{3}$.

17. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function and P be a polynomial of degree 4 with coefficients in $\mathbb{R}$. If $P(f(x)) = 0$ for all $x \in \mathbb{R}$, then

   (a) $f(x) = 0$ for all $x \in \mathbb{R}$.
   (b) f is a constant function.
   (c) for all continuous functions g, there exists $x \in [0, 1]$ such that $P(g(x)) = 0$.
   (d) P has at most two roots which do not belong to $\mathbb{R}$.